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PHYSICS, CHEMISTRY AND APPLICATION OF NANOSTRUCTURES

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SPIN DYNAMICS IN SEMICONDUCTOR NANOSTRUCTURES

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We review our theoretical investigations on the spin relaxation/dephasing in spin precession and spin diffusion/transport in semiconductor nanostructures based on the kinetic spin Bloch equation approach.

1. Introduction

Much attention has been devoted to the electron spin dynamics in semiconductors for the past three decades [1,2]. Especially, recent experiments have shown extremely long spin lifetime (up to hundreds of nanoseconds) in *n*-type bulk zinc-blende semiconductors (such as GaAs) [4-5]. Moreover, more investigations have been performed on various low dimensional systems [6]. The spin diffusion/transport has also been studied experimentally, and very long spin injection length are reported [7]. These findings show the great potential for using the spin degree of freedom in place of, or in addition to, the charge degree of freedom for device application such as qubits and spin transistors. A thorough understanding of the spin relaxation/dephasing (R/D) in the spin precession and spin diffusion/transport is essential for such applications.

It is understood that the D'ayakonov-Perel' (DP) mechanism is the leading spin R/D mechanism in *n*-type zinc-blende semiconductors [8]. Many theoretical works have been carried out to study the spin relaxation time in various systems [1,9] based on the single-particle formula [1]

$$\frac{1}{\tau} = \int_0^\infty dE_k \left(f_{k\uparrow} - f_{k,\downarrow} \right) \tau_p(k) \overline{\boldsymbol{h}^2(\boldsymbol{k})} \Bigg/ 2 \int_0^\infty dE_k \left(f_{k\uparrow} - f_{k\downarrow} \right). \tag{1}$$

Here $\tau_p(k)$ is the momentum relaxation time which is due to the electron-phonon and electron-impurity scattering, $f_{k\sigma}$ stands for the electron distribution functions of spin σ, $\boldsymbol{h}(\boldsymbol{k})$ is the DP term which serves as an effective magnetic field and is composed of the Dresselhaus term [10] due to the bulk inversion asymmetry (BIA) and the Rashba term [11] due to the structure inversion asymmetry (SIA),

$\overline{h^2(\boldsymbol{k})}$ denotes the average of $h^2(\boldsymbol{k})$ over all directions of $\boldsymbol{k}$. In GaAs quantum well (QW), the Dresselhaus term is dominant and $\boldsymbol{h}(\boldsymbol{k})$ has the form:

$$h_x(\boldsymbol{k}) = \gamma\, k_x (k_y^2 - \langle k_z^2 \rangle), h_y(\boldsymbol{k}) = \gamma\, k_y (\langle k_z^2 \rangle - k_x^2), h_z(\boldsymbol{k}) = 0, \qquad (2)$$

in which $\langle k_z^2 \rangle$ represents the average of the operator $(\partial/\partial z)^2$ over the electronic state of the lowest subband, γ is the Dresselhaus spin-orbit parameter [1,12]. For InAs, the Rashba term is more important and $\boldsymbol{h}(\boldsymbol{k})$ is given by $h_x(\boldsymbol{k}) = \alpha k_y$, $h_y(\boldsymbol{k}) = -\alpha k_x$ and $h_z(\boldsymbol{k}) = 0$, in which the Rashba coefficient α is proportional to the interface electric field E_z along the growth direction: $\alpha = \alpha_0 E_z$, with the coefficient α_0 being inversely proportional to the energy gap and the effective mass [13]. Equation (1) is valid only when $|\boldsymbol{h}|\, \tau_p \ll 1$, *i.e.*, the strong scattering limit, and the scattering is elastic. It also cannot be applied to a system far away from the equilibrium, such as a system with large spin polarization and/or with a strong in-plane electric field. Moreover, the Coulomb scattering has long been neglected as it does not contribute to the momentum relaxation directly.

It was shown recently by Wu *et al.* from a full microscopic kinetic-spin-Bloch-equation (KSBE) approach that the single-particle approach is inadequate in accounting for the spin R/D both in the time domain [14-18] and in the space domain [19-22]. The momentum dependence of the effective magnetic field (the DP term) and the momentum dependence of the spin diffusion rate along the spacial gradient [19] or even the random spin-orbit coupling (SOC) [23] all serve as inhomogeneous broadenings [15,16]. It was pointed out that in the presence of inhomogeneous broadening, any scattering, including the carrier-carrier Coulomb scattering, can cause an irreversible spin R/D [15,16]. Moreover, besides the spin R/D channel the scattering provides, it also gives rise to the counter effect to the inhomogeneous broadening. The scattering tends to drive carriers to a more homogeneous state and therefore suppresses the inhomogeneous broadening. Finally, this approach is valid in both strong and weak scattering limits and can be used to study systems far away from the equilibrium.

In this paper, we review the KSBE approach in various nanostructures and under different conditions.

2. Model and KSBEs

We start our investigation from an *n*-type zinc-blende semiconductor QW with the growth direction along the *z*-axis. The well width is assumed to be small

enough so that only the lowest subband is relevant. Sometimes a moderate magnetic field B is applied along the x-axis (in the Voigt configuration).

By using the nonequilibrium Green function method with gradient expression as well as the generalized Kadanoff-Baym Ansatz [24], we construct the KSBEs as follows:

$$\dot{\rho}_k(\boldsymbol{r},t) = \dot{\rho}_k(\boldsymbol{r},t)|_{dr} + \dot{\rho}_k(\boldsymbol{r},t)|_{dif} + \dot{\rho}_k(\boldsymbol{r},t)|_{coh} + \dot{\rho}_k(\boldsymbol{r},t)|_{scat}. \tag{3}$$

Here $\rho_k(\boldsymbol{r},t) = \begin{pmatrix} f_{k\uparrow} & \rho_{k\uparrow\downarrow} \\ \rho_{k\downarrow\uparrow} & f_{k\downarrow} \end{pmatrix}$ are the density matrices of electrons with momentum $\boldsymbol{k}$ at position $r = (x,y)$ and time t. The off-diagonal elements $\rho_{k\uparrow\downarrow} = \rho^*_{k\uparrow\downarrow}$ represent the correlations between the spin-up and -down states, $\dot{\rho}_k(\boldsymbol{r},t)|_{dr} = \{\nabla_r\overline{\varepsilon_k}(\boldsymbol{r},t), \nabla_k\rho_k(\boldsymbol{r},t)\}/2$ are the driving terms from the external electric field. Here $\overline{\varepsilon_k}(\boldsymbol{r},t) = \boldsymbol{k}^2/2m^* + [g\mu_B\boldsymbol{B} + \boldsymbol{h}(\boldsymbol{k})]\cdot\boldsymbol{\sigma}_{\sigma\sigma'}/2 - e\Psi(\boldsymbol{r}) + \varepsilon_{HF}(\boldsymbol{r},t)$ with $\varepsilon_{HF}(\boldsymbol{r},t) = -\sum_q V_q\rho_{k\text{-}q}(\boldsymbol{r},t)$ representing the Hartree-Fock (HF) term, m^* is the effective mass. $\Psi(\boldsymbol{r})$ is determined from the Poisson equation. The bracket $\{A,B\} = AB + BA$ is the anti-commutator. $\dot{\rho}_k(\boldsymbol{r},t)|_{dif} = -\{\nabla_k\overline{\varepsilon_k}(\boldsymbol{r},t), \nabla_r\rho_k(\boldsymbol{r},t)\}/2$ represent the diffusion terms. The coherent terms in Eq. (3) are $\dot{\rho}_k(\boldsymbol{r},t)|_{coh} = i[[g\mu_B\boldsymbol{B} + \boldsymbol{h}(\boldsymbol{k})]\cdot\boldsymbol{\sigma}_{\sigma\sigma'}/2 - e\Psi(\boldsymbol{r}) + \varepsilon_{HF}(\boldsymbol{r},t), \rho_k]$.

The scattering terms are different depending on different statistics [25]. One is the collinear statistics where the equilibrium state is taken as the Fermi distribution of electrons in the conduction band without the SOC term (DP term). Therefore the energy spectrum is $\varepsilon_k(\boldsymbol{r},t) = k^2/2m^*$ and the eigenstates of spin are the eigenstates of σ_z, i.e. $\chi_\uparrow = (1,0)^T$ and $\chi_\downarrow = (0,1)^T$. The other is the helix statistics where the equilibrium state is taken to be the Fermi distribution of electrons in the conduction band with the SOC. The energy spectrum is then $\varepsilon_{k,\xi}(\boldsymbol{r},t) = \boldsymbol{k}^2/2m^* + \xi|\boldsymbol{h}(\boldsymbol{k})|$ with $\xi = \pm 1$ for the two spin branches. The eigenfunctions of spin are therefore $|\xi\rangle = [\chi_\uparrow + \xi\tilde{h}(\boldsymbol{k})\chi_\downarrow/|\boldsymbol{h}(\boldsymbol{k})|]/\sqrt{2}$ with $\tilde{h}(\boldsymbol{k}) = h_x(\boldsymbol{k}) + ih_y(\boldsymbol{k})$. In principle, the helix statistics is the correct statistics. Nevertheless, as the SOC is very small in semiconductors compared to the Fermi energy, the SOC in the energy spectrum can be neglected [25]. This also facilitates an accurate numerical calculation of scattering.

The spin R/D times can be determined from the time evolution of the density matrix ρ_k by numerically solving the KSBEs with all the scattering explicitly included. The spin relaxation time T_1 is determined from the slope of the envelope of $\Delta N = \sum_k (f_{k\uparrow} - f_{k\downarrow})$. The irreversible spin dephasing time T_2 is

associated with the incoherently-summed spin coherence [14] $\rho = \sum_k |\rho_{k\uparrow\downarrow}(t)|$ whereas the ensemble spin dephasing time T_2^* is defined from the slope of the envelope of the coherently-summed spin coherence $\rho' = |\sum_k \rho_{k\uparrow\downarrow}(t)|$. Similarly, for spin diffusion/transport, the spin diffusion length can be determined from the spacial evolution of ΔN in the steady state [19].

3. Spin R/D

In this section we review our results of spin R/D in semiconductor nanostructures based on SKBE approach. It was shown that any spin conserving scattering, including the electron-electron Coulomb scattering, can cause an irreversible spin R/D in the presence of the inhomogeneous broadening [15-18].

The energy dependence of the *g* factor [15] and the momentum dependence of the DP term [16-18] all serve as the inhomogeneous broadening. In a quantum wire, the inhomogeneous broadening can be quantified by the standard deviation of the Larmor frequencies which well corresponds to the resulting spin dephasing time [26]. It is further shown in GaAs (110) QWs that in the presence of a magnetic field in the Voigt configuration, there is inhomogeneous broadening from the DP term and the spin R/D time is finite, although much longer than that in (001) QWs [27].

A thorough many-body investigation on *n*-type GaAs (001) QWs is performed with all the scattering explicitly included [17] when the temperature is higher than 120 K. It is shown that in QW with small well width, the spin R/D time increases with the temperature in stead of decreases as predicted from the single-particle approach when the electron density is in the order of 10^{11} cm^{-2}. This temperature dependence is in a good agreement with the experimental result by Malinowski *et al*. [6]. For larger well width, the situation may become different. Weng and Wu calculated the spin R/D for larger well widths by including the multi-subband effect [28]. It is shown that for small/large well width so that the linear/cubic term in Eq. (2) is dominant, the spin R/D time increases/decreases with the temperature. This is because with the increase of temperature, both the inhomogeneous broadening and the scattering get enhanced.

The relative importance of these two competing effects is different when the linear/cubic term is dominant [28]. As the Coulomb scattering is included, one is able to calculate the spin R/D far away from the equilibrium. It is shown in [17] that the spin R/D time increases dramatically with the spin polarization. This is discovered due to the lowest order (HF) contribution of the Coulomb

interaction which appears in the coherent term of the KSBEs. With high spin polarization, the HF term serves as an effective magnetic field along the *z* axis which blocks the spin precession. It is further shown in [18] that one is able to calculate the spin R/D in the presence of a high in-plane electric field so that the system is in the hot-electron regime.

In the presence of an in-plane electric field, electron spin can precess in the absence of any magnetic field at high temperature. This is understood that the in-plane electric field induces a center-of-mass shift of the momentum which gives rise to an effective magnetic field proportional to the electric field [18]. The effect of strain on the spin R/D is also discussed and it is shown that one can effectively manipulate the spin R/D time by strain [29]. Cheng and Wu further discussed the spin R/D under identical Dresselhaus and Rashba terms [25]. A finite spin R/D time is obtained due to the cubic term in Eq. (2).

Very recently, Zhou *et al*. [30] extended the theory to the regime of very low temperature by figuring out ways to deal with the electron-AC phonon scattering numerically. An excellent agreement with experiment by Ohno *et al*. [6] is obtained from 20 K to 300 K without any fitting parameter. The Dresselhaus coupling parameter γ is found to be well represented by $\gamma = (4/3)\left(m^*/m_0\right)\left(1/\sqrt{2m^{*3}E_g}\right)\left(\eta/\sqrt{1-\eta/3}\right)$ [12] with m_0 denoting the free electron mass, E_g being the band gap and $\eta = \Delta/\left(\Delta + E_g\right)$, Δ is the SOC of the valence band.

Moreover, a footprint of the Coulomb scattering on the spin R/D is predicted at low temperature when the electron density is not too high and the impurity density is low [30]. Lü *et al.* applied the KSBEs to study the heavy and light holes in (001) GaAs QWs [31] where the SOC is due to the Rashba term [32] and is very strong so that the system is in the weak scattering limit. Therefore the single-particle formula (1) fails and only the KSBE approach is applicable. It is shown that in the weak scattering limit, adding a new scattering, such as the Coulomb scattering provides an additional spin R/D channel so that the spin R/D time is shorter [31]. This is in opposite to the case of the strong scattering limit [18,31]. Finally, it is pointed out in [33] that due to the strong Coulomb scattering, $T_1=T_2=T_2^*$ is valid over a very wide temperature and density regime.

4. Spin diffusion/transport

By solving the KSBEs together with the Poisson equation self-consistently, one is able to obtain all the transport properties such as the mobility, charge diffusion length and spin diffusion/injection length without any fitting

parameter. It was first pointed out by Weng and Wu [19] that the drift-diffusion equation approach is inadequate in accounting for the spin diffusion/transport. It is important to include the off-diagonal term $\rho_{k\uparrow\downarrow}$ in studying the spin diffusion/transport. With this term, electron spin precesses along the diffusion and therefore $\boldsymbol{k} \bullet \nabla_r \rho_k(\boldsymbol{r}, t)$ in the diffusion term offers an *additional* inhomogeneous broadening. With this additional inhomogeneous broadening, any scattering, including the Coulomb scattering, can cause an irreversible spin R/D [19]. Unlike the spin precession in the time domain where the inhomogeneous broadening is determined by $\boldsymbol{h}(\boldsymbol{k})$, here it is determined by $|g\mu_B\boldsymbol{B}+\boldsymbol{h}(\boldsymbol{k})|/k_x$ provided the diffusion along the x-axis. Therefore, even in the absence of the DP term, the magnetic field alone can provide an inhomogeneous broadening [19].

Moreover, it is first pointed out that a spin pulse can oscillate along the diffusion in the absence of the magnetic field at very high temperature [20]. Detailed study is performed later on this effect [21,22]. This oscillation was later realized experimentally by Crooker and Smith [7] in bulk system at very low temperature. It is also shown that the Coulomb drag plays a very important role in the spin diffusion and the electric field can enhance/suppress the spin diffusion [22]. Very recently Cheng and Wu developed a new numerical scheme to calculate the spin diffusion/transport [34] with very high accuracy and speed. It is discovered that due to the scattering, especially the Coulomb scattering, $T_2=T_2^*$ is valid even in the space domain. Moreover, as the inhomogeneous broadening in spin diffusion is determined by $|\boldsymbol{h}(\boldsymbol{k})|/k_x$ in the absence of magnetic field, the period of the spin oscillations along the x-axis is independent on the electric field [34] which is different from the spin precession rate in the time domain [18]. This is consistent with the experimental findings by Beck *et al.* [7]. Many properties of the spin diffusion/transport in the steady state are addressed in detail in [34].

5. Summary

We reviewed our fully microscopic KSBE study of spin R/D in the spin precession and spin diffusion/transport. The importance of the Coulomb scattering is stressed. This approach allows one to deal with systems both near and far away from the equilibrium and in both the strong and weak scattering limits. Moreover, both the effect of inhomogeneous broadening on the scattering and the counter effect of the scattering on the inhomogeneous broadening are fully accounted.

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